

For publication in the AIAA Journal
as a Technical Note

(NASA TM 1-51370)

ENTROPY PRODUCTION IN VIBRATIONAL NONEQUILIBRIUM NOZZLE FLOW

By

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N65-89021

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Code 2A

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It is well known that the existence of nonequilibrium in a flow system serves as a source of internal entropy generation. This note is concerned with the entropy production resulting from vibrational nonequilibrium in a hypersonic nozzle and is based on the numerical calculations presented in reference 1.

That work presented a quasi-one-dimensional analysis of the vibrational-nonequilibrium flow of nitrogen in a hypersonic nozzle having a geometry given by

$$\frac{A}{A^*} = 1 + \left(\frac{\tan \theta}{r^*} \right)^2 x^2 \quad (1)$$

where A = nozzle cross-sectional area, A^* = throat area, θ = nozzle half angle, r^* = nozzle throat radius, and x = distance from throat along nozzle axis.

Equilibrium was assumed to exist from the stagnation chamber to the nozzle throat and the vibrational relaxation during the remainder of the expansion was governed by the rate equation

$$\frac{d\sigma}{dt} = \frac{\sigma_e - \sigma}{\tau} \quad (2)$$

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where σ = vibrational energy, the subscript e denotes equilibrium, and the vibrational-relaxation time for nitrogen, τ_p , is given by

$$\tau_p = AT^{1/2} \exp\left(\frac{1}{B/T}\right) \quad (2a)$$

where the constants A and B are those obtained in reference 2 by an empirical correlation of data.

The degree to which the flow parameters in the nozzle are affected by the existence of nonequilibrium was shown. In order to show these effects as a function of initial stagnation pressure, p_0 , and nozzle size and geometry, the correlating group $L = p_0 \frac{r^*}{\tan \theta}$ in units of atm-cm was varied in that investigation. (For a representative case where $r^* = 0.125$ cm, $\theta = 10^\circ$, and $p_0 = 1000$ atm, $L = 709$.)

In a review of reference 1, Presley (ref. 3) suggested that the work be extended to include the calculation of the entropy production in the nozzle flow process. This note deals with such a calculation. In the present study the following model is used to describe the gas system in vibrational nonequilibrium. The energy modes are divided into two subsystems. The first of these contains only the translational and rotational energy modes. Thus the temperature of this subsystem is the translational temperature T. The second subsystem represents the vibrational energy and is assumed to be composed of harmonic oscillators with a Boltzmann distribution with respect to energy levels. Thus a vibrational temperature, T_v , can be defined by the relation

$$\sigma = \frac{R\theta_v}{\exp\left(\frac{\theta_v}{T_v}\right) - 1} \quad (3)$$

where θ_V = characteristic temperature of molecular vibration and R = gas constant. By considering the energy exchange between the two subsystems it can be shown (ref. 4) that the internal entropy generation due to vibrational nonequilibrium is given by

$$d_i s = \left(\frac{1}{T_V} - \frac{1}{T} \right) d\sigma \quad (4)$$

Now consider equation (4) with relation to inviscid flow in a hypersonic nozzle. Throughout this expansion process $d\sigma \leq 0$. For the two limiting cases of equilibrium flow and frozen flow, the expansion will be isentropic, since for the equilibrium case $T_V = T$ and for the frozen case $d\sigma = 0$.

For the intermediate case considered in reference 1, the flow from the stagnation chamber to the throat was assumed to be in equilibrium, that is $T = T_V$, so that in this initial phase of the expansion $d_i s = 0$. Then as further expansion occurs beyond the throat, vibrational nonequilibrium effects can come into play. These occur when rate of vibrational adjustment is too slow to allow the vibrational temperature, T_V , to decrease as rapidly as the translational temperature T . It can be seen from equation (4) that this must result in an increase in entropy. As the expansion proceeds, the pressure and temperature of the gas continually decrease resulting in a corresponding increase in relaxation time, τ . Consequently, $\frac{d\sigma}{dt}$ decreases and eventually becomes negligible so that further expansion occurs isentropically.

To determine the entropy increase from the stagnation chamber to any position l in the nozzle, equation (4) can be integrated numerically from $x = 0$ to $x = l$.

$$\Delta \frac{s_i}{R} = \frac{1}{R} \int_0^l \left(\frac{1}{T_V} - \frac{1}{T} \right) \frac{d\sigma}{dx} dx \quad (5)$$

Using equation (5), the entropy production in vibrational nonequilibrium flow of nitrogen was evaluated for a single stagnation condition, $p_0 = 100$ atm, $T_0 = 4,000^\circ$ K, and a range of values of L . The values of T , T_V , and $\frac{d\sigma}{dx}$ versus x needed to carry out the integration were obtained from the numerical computations which were made for reference 1. The results obtained by numerical integration are shown in figure 1. The line of constant stagnation enthalpy in the upper portion of the figure is based on equilibrium relationships. This is consistent with the assumption of equilibrium in the stagnation chamber and with the definition of a total pressure. The remainder of the plot simply shows the entropy variation as a function of pressure ratio during the expansion process for various values of L .

The limiting cases of equilibrium ($L = \infty$) and frozen flow ($L = 0$) appear as vertical lines on figure 1, since such expansions are isentropic. In each case involving nonequilibrium the expansion proceeds initially in equilibrium up through the throat with no entropy change, then goes through a region of nonequilibrium during which entropy is produced, and finally freezes with no further entropy increase. It should be

noted that for small values of L , the entire flow is nearly frozen and the total entropy increase is small. As L is increased to intermediate values the portion of the flow in which nonequilibrium exists increases and thus the entropy produced is greater. For large values of L the flow is closer to equilibrium and the total entropy produced is less. (For the lowest values of L , the possibility exists of the flow freezing upstream of the throat and altering the results somewhat. However, the trend indicated by the present analysis should still apply.)

An appreciation of the consequences of entropy production in the nozzle can be gained by considering its effect on free stream total pressure, $p_{o,1}$. The free stream total pressure is defined as the pressure which would exist if the flow at free stream conditions could be compressed isentropically to an equilibrium state with a zero velocity (i.e., to the total enthalpy based on equilibrium). This imaginary compression could be thought of as taking place by the following two-step process. Starting from a nonequilibrium state in the free stream, the gas first undergoes a frozen isentropic compression ($d\sigma = 0$) to the point at which $T = T_v$. The remainder of the compression is carried out isentropically in equilibrium ($T = T_v$).

The free stream total pressure, $p_{o,1}$, can be determined from figure 1 for any free stream condition by following a constant entropy path up to the line of constant stagnation enthalpy. The points indicated on the line of constant stagnation enthalpy show the maximum reduction in $p_{o,1}$ for the various values of L . For the conditions investigated, the loss in free stream total pressure due to nonequilibrium can be as great as 4 percent as shown for the case $L = 500$.

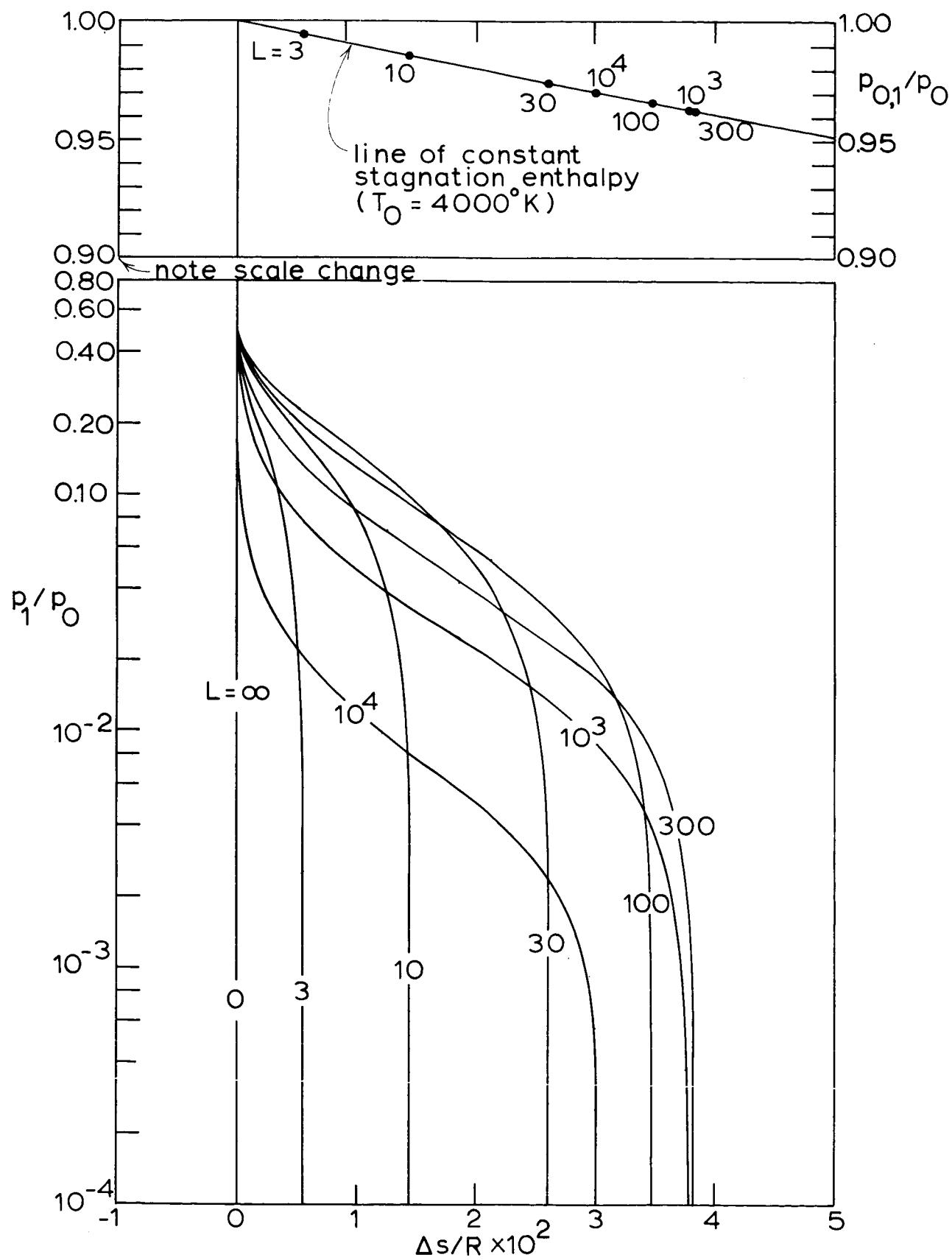
Additional calculations were made to compare the pitot pressure, p_0' , of a nonequilibrium nozzle flow to that of an equilibrium nozzle flow with identical stagnation conditions and area ratios. In these calculations equilibrium flow was assumed to exist behind the shock. It was found that a lower pitot pressure, p_0' , is always observed for the nonequilibrium case. This drop in p_0' is of somewhat smaller proportions than the drop in $p_{0,1}$ but no explicit relationship between the two is available.

However, the free stream conditions at any point along the nozzle axis can be calculated as was done in reference 1 and all entropy production, or nonequilibrium effects, in the nozzle will be reflected in these free stream quantities. Using these free stream values and applying the conservation equations across the shock, the properties behind the shock can be calculated using a simple iterative procedure. In addition, if the flow behind the shock is assumed to be in equilibrium, the pitot pressure can easily be calculated.

REFERENCES

1. Erickson, W. D.: Vibrational-Nonequilibrium Flow of Nitrogen in Hypersonic Nozzles. NASA TN D-1810 (1963).
2. Treanor, C. E., and Marrone, P. V.: The Effect of Dissociation on the Rate of Vibrational Relaxation. Rep. No. QM-1626-A-4, Cornell Aero. Lab. Inc., (Feb. 1962).
3. Presley, L. L.: Private Communication, Ames Research Center (Jan. 1963 and June 1963).
4. Vincenti, W. G.: Lectures on Physical Gas Dynamics. Stanford University (1961).

Figure 1.- Entropy production as a function of free-stream pressure for various values of the parameter L.



Connor, Erickson Figure 1